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Note

Thickness and refractive index measurements using multiple beam interference fringes (FECO)

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Abstract

We report on the use of optical interferometry employing fringes of equal chromatic order (FECO) in a surface force apparatus (SFA) to determine film thicknesses and refractive indices of confined media for a wide range of separations. In particular, we show how to calculate the surface separation (film thickness) based on two fringes whose contact position was not measured. We discuss the measurement accuracy, and though the theoretical accuracy is 1 Å for all separations, we show that in practice, for large separations, it is very hard to get to this accuracy.

Keywords: Large separations; Thick films; Three-layer interferometer; Dispersion; Dispersive phase change

In a SFA (surface force apparatus) [1] experiment, there is usually a three- or five-layer interferometer [2]. For simplicity we assume here a three-layer interferometer, although the analysis applies to any number of layers. When the transparent substrate layers (typically mica sheets) are brought into contact, the positions λ_p^0 of the pth-order fringes is usually measured in the visible range of wavelengths. After the substrate surfaces are separated by a distance D, a three-layer interferometer is formed. The fringes shift to longer wavelengths and their new positions λ_p^D are given (see Ref. [3]) by

$$\tan\left(2\pi\mu_{\text{med}}D/\lambda_p^D\right) = \frac{2\bar{\mu}\sin(4\pi\mu Y/\lambda_p^D)}{(\bar{\mu}^2-1)-(1+\bar{\mu}^2)\cos(4\pi\mu Y/\lambda_p^D)}, \tag{1}$$

which, assuming no dispersion (we discuss this point later in the text), reduces to the more useful form

$$\tan\left(2\pi\mu_{\text{med}}D/\lambda_p^D\right) = \frac{2\bar{\mu}\sin\left(\pi\frac{1-\lambda_p^0/\lambda_p^D}{1-\lambda_p^0/\lambda_{p-1}^0}\right)}{(1+\bar{\mu}^2)\cos\left(\pi\frac{1-\lambda_p^0/\lambda_p^D}{1-\lambda_p^0/\lambda_{p-1}^0}\right)\pm(\bar{\mu}^2-1)}, \tag{1a}$$

where Y is the optical thickness of each substrate (we discuss later the difference between the optical and physical thicknesses) and $\bar{\mu}=\mu/\mu_{\text{med}}$, where μ (μ_γ or μ_β) and μ_{med} are, respectively, the refractive indices of the mica and the intervening medium at λ_p^D. In Eq. (1a), $+$ and $-$ refer to p odd and p even order fringes, respectively. Other ways of calculating thicknesses are available (see, for example, Ref. [4]); however, the simplicity of Eq. (1a) made it the most common method. Indeed, due to the increased use of Eq. (1a) in SFA measurements, people have addressed some special cases which were not addressed by the original publication [3], such as reflecting (metallic) media [5], non-symmetric three-layer interferometry [6], and most recently the splitting of the fringes as a result of a liquid crystalline medium [7]. Currently Eq. (1a) is limited due to the fact that the fringes at contact need to be of the same order as the fringes of measurements. Apart from imposing inconvenience, this also limits the range of measurable separations, since at far separations, the contact fringes are no longer in the visible range. Different approaches to this problem were addressed by Horn and Smith [6], by Heuberger et al. [4], and by Farrell et al. [8]. In this study, however, we present a very simple account for this problem. An even simpler account than the one presented here is given in the thesis of H. Christenson [9]. This solution, however, is extremely sensitive to the position of the fringes, sub-Ångstrom errors in the wavelength measurement can produce a micrometer-scale

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error in the separation. This paper describes an easy and accurate way of calculating the separation from any order of fringes at any surface separation.

From Eq. (1a) it is clear that if the refractive indices of the (mica) substrate and the medium are known then the separation between the surfaces D can be obtained from λ_p^D, λ_p^0, and λ_{p-1}^0. In the case where the medium refractive index μ is not known [10,11], there is a need to solve two equations with two unknowns, D and μ_{med}. The two equations are Eq. (1a) for an odd fringe and Eq. (1a) for an even fringe. In order to solve these simultaneous equations, we need to know the location of two adjacent fringes, λ_p^D and λ_{p-1}^D, at finite D and the "contact" positions of three adjacent fringes, λ_p^0, λ_{p-1}^0, and λ_{p-2}^0, at $D = 0$.

The limitation of the above "conventional" method is that λ_p^D, λ_{p-1}^D, λ_p^0, λ_{p-1}^0, and λ_{p-2}^0 need to be in the visible for them to be measured. This limits the range of measurable separations D obtained using this approach to $D < 1$ µm for typical substrate thicknesses (thicker substrates would increase the range but decrease the accuracy of the measurements).

In this paper we describe a procedure that allows accurate and unambiguous measurements of distance and refractive index to be made using the contact position of only two adjacent fringes, λ_p^0 and λ_{p-1}^0, and—more importantly—using two adjacent fringes of *any* order, λ_m^D and λ_{m-1}^D, and not necessarily λ_p^D and λ_{p-1}^D.

1. General procedure for measuring D and μ_{med} of films of arbitrary thickness

The "contact" positions, λ_p^0, correspond to a single-layer interferometer. A one-layer interferometer made of a substrate with a refractive index μ and physical thickness Y', obeys the relation

$$2\mu Y' = p\lambda, \quad p = 1, 2, 3, \ldots, \infty, \tag{2}$$

where λ is the wavelength that corresponds to constructive interference and p is a natural number (the fringe order). In practice there is never a one-layer interferometer, since the reflecting layer results in a phase change of the light at the substrate–reflector interface (say a mica–silver interface). Such a phase change for the mica–silver interface reflection has been studied [12], and it has been shown that it can be viewed as an apparent small change in the optical thickness of the substrate to Y instead of Y' [13]:

$$2\mu Y = p\lambda, \quad p = 1, 2, 3, \ldots, \infty. \tag{2a}$$

It is intuitive that the solution to the proposed problem may be provided using this equation. Specifically, using Eq. (2a) to substitute the thickness of the layer Y for the contact position of the fringe in Eq. (1a), we should be able to relate one contact fringe position to any other contact fringe position. Hence, the problem becomes one of how to express a single-layer interferometer in terms of a relation between different fringe orders (rather than the presentation of Eq. (2a)).

Using Eq. (2a) and assuming constant Y [13], we write for three arbitrary fringes of order p, $p-1$, and m

$$2\mu_p Y = p\lambda_p^0, \tag{3a}$$

$$2\mu_{p-1} Y = (p-1)\lambda_{p-1}^0, \tag{3b}$$

$$2\mu_m Y = m\lambda_m^0, \tag{3c}$$

where we write μ_p to emphasize the dispersive nature of the substrate, i.e., that each refractive index μ_p corresponds to a wavelength λ_p^0. Equations (3a) and (3b) have just two unknowns, Y and p, and solving for them we get

$$\frac{1}{p} = 1 - \frac{\mu_{p-1}\lambda_p^0}{\mu_p\lambda_{p-1}^0}. \tag{4}$$

From Eqs. (3a) and (3c) and using Eq. (4) we obtain

$$\lambda_m^0 = \frac{\mu_m}{\mu_p} \frac{\lambda_p^0}{1 - (p-m)\left(1 - \frac{\mu_{p-1}\lambda_p^0}{\mu_p\lambda_{p-1}^0}\right)}. \tag{5}$$

Equation (5) relates the position of the mth fringe to that of the pth fringe as a function of the number of fringes $(p-m)$ between them, as well as five other parameters that need to be known, including μ_m. Note that $(p-m)$ is an integer, which may be positive, negative, or zero. Now, we may use any fringe in order to use Eq. (1a) for the calculation of separations and refractive indices: we simply need to count the number of fringes from m to p, put this number as $(p-m)$ into Eq. (5), and calculate a new set of contact positions λ_m^0, λ_{m-1}^0, and λ_{m-2}^0. As we show later in the worked Example 1, the value of μ_m can be obtained by substitution of an approximated λ_m given by Eq. (6) into Eq. (12), which we introduce below. Experimentally, if a measurement is done at a large separation on a fringe λ_m^D whose contact position λ_m^0 is not visible, one way of finding $(p-m)$ (after completing a measurement using the mth fringe) is to bring the surfaces to substrate–substrate contact while counting the passing fringes until one finds the p-fringe. This number is the integer $(p-m)$.

Equation (5) requires knowledge of μ_m. To obtain this, note that Eq. (5) has a very weak dependence on the refractive index of the substrate. Neglecting the dispersion, for example, results in

$$\lambda_m^0 = \frac{\lambda_p^0}{1 - (p-m)(1 - \lambda_p^0/\lambda_{p-1}^0)}, \tag{6}$$

which is an excellent approximation for many cases. Equation (6) can also be used for a first guess for the wavelength at which one needs to look for μ_m in Eq. (5). In worked Example 1, we show how this can be done. Similarly, Eq. (5) or its approximated version, Eq. (6), which is written for any λ_m^0, can also give λ_{p-2}^0, thus reducing the minimal number of required contact fringes to 2. In fact, there are cases where only two fringes are in the visible range (for very thin mica), for which Eq. (5) may be very useful. In worked Example 2 we show that for a three-layer interferometer of known medium refractive index and substrate dispersion [14], one can theoretically calculate the positions of the contacting fringes even without counting the integer $(p - m)$. However, in practice this can be used only for small $(p - m)$; therefore, for large separations, $(p - m)$ usually needs to be counted from the passing fringes as described above.

The simplified result of Eq. (6) can also be obtained by putting $D = 0$ in Eq. (1a). This gives

$$\frac{1 - \lambda_p^0/\lambda_p^D}{1 - \lambda_p^0/\lambda_{p-1}^0} = z, \tag{7}$$

where z is an integer. In Eq. (7), λ_p^D is not the contact position λ_p^0, but *any* contact position λ_m^0. This is because any such fringe would correspond to $\tan(2\pi\mu_{\mathrm{med}}D/\lambda_p^D) = 0$ in Eq. (1a). Thus, solving for Eq. (7), we again get Eq. (6).

It is clear from the above treatment that Eq. (1a) does not take into account the dispersion of the substrate—as was noted already in the original paper [3]. The dispersion of mica has been studied [15,16], and below we derive the equation corresponding to Eq. (1) for dispersive substrates and media.

Using the more fundamental form of Eq. (1), rather than (1a), we note that a one-layer interferometer implies that

$$\frac{4\pi\mu Y}{\lambda_p^D} = p\pi\left(\lambda_p^0/\lambda_p^D\right), \tag{8}$$

where μ is the refractive index of the mica at λ_p^D (as noted in Eq. (1)). Substituting Eqs. (4) and (8) into Eq. (1), we obtain the dispersive version of (1a) as

$$\begin{aligned}\tan&\left(2\pi\mu_{\mathrm{med}}D/\lambda_p^D\right)\\ &= 2\bar{\mu}\sin\left(\pi\left(1-\lambda_p^0/\lambda_p^D\right)\left(1-\frac{\mu_{p-1}\lambda_p^0}{\mu_p\lambda_{p-1}^0}\right)^{-1}\right)\\ &\quad\times\left[(1+\bar{\mu}^2)\cos\left(\pi\left(1-\lambda_p^0/\lambda_p^D\right)\left(1-\frac{\mu_{p-1}\lambda_p^0}{\mu_p\lambda_{p-1}^0}\right)^{-1}\right)\right.\\ &\qquad\left.\pm(\bar{\mu}^2-1)\right]^{-1}.\end{aligned} \tag{9}$$

Equation (9) is the analogue of Eq. (1a) with a correction for the dispersion in the substrate refractive index, and as in Eq. (1a), $+$ and $-$ refer to p odd and p even order fringes, respectively.

Finally, we may write the analogue to Eq. (9) for any order of fringe,

$$\begin{aligned}\tan&\left(2\pi\mu_{\mathrm{med}}D/\lambda_m^D\right)\\ &= 2\bar{\mu}\sin\left(\pi\left(1-\lambda_m^0/\lambda_m^D\right)\left(1-\frac{\mu_{m-1}\lambda_m^0}{\mu_m\lambda_{m-1}^0}\right)^{-1}\right)\\ &\quad\times\left[(1+\bar{\mu}^2)\cos\left(\pi\left(1-\lambda_m^0/\lambda_m^D\right)\left(1-\frac{\mu_{m-1}\lambda_m^0}{\mu_m\lambda_{m-1}^0}\right)^{-1}\right)\right.\\ &\qquad\left.\pm(\bar{\mu}^2-1)\right]^{-1},\end{aligned} \tag{10}$$

and then use Eq. (5) to replace the unknowns λ_m^0 and λ_{m-1}^0 with the measured λ_p^0 and λ_{p-1}^0 (and the calculated λ_{p-2}^0 in the case where the refractive index of the medium is not known).

In certain situations, the integer number $(m - p)$ may not be available or measurable. Since we have two different equations (10) for odd and even fringes, we can use the measured λ_m^D and λ_{m-1}^D, and simply guess a value for $(p - m)$. Then, using Eq. (5) with λ_p^0 and λ_{p-1}^0, we calculate the contact positions which correspond to fringes m, $m - 1$, and $m - 2$. Apparently, only if the guess is correct will one obtain the same separation D using both forms of Eq. (10) for even and for odd fringes. In practice, however, this method (which anyway works only for a three-layer interferometer) can be done only for very small $(p - m)$ values, while for large $(p - m)$ other factors such as the dispersion of the medium μ_{med} and the dispersive phase change at the reflector–substrate interface should also be known very accurately. This is discussed further after Example 2. The examples below are based on measured values.

2. Worked examples

2.1. Example 1

Estimating the value of λ_{p-2}^0 from λ_p^0 and λ_{p-1}^0.

In an experiment using "brownish" mica as substrate, the contact wavelengths of three adjacent fringes of unknown order p, $p - 1$, and $p - 2$ are measured (Fig. 1) and found to be at $\lambda_p^0 = 5604.18 \pm 0.11$ Å, $\lambda_{p-1}^0 = 5761.07 \pm 0.16$ Å, and $\lambda_{p-2}^0 = 5927.38 \pm 0.21$ Å. If we want to *estimate* the position of λ_{p-2}^0 from λ_p^0 and λ_{p-1}^0, then putting $p - m = 2$ into Eq. (6) we obtain

$$\begin{aligned}\lambda_{p-2}^0 &= \frac{5604.18}{1 - 2(1 - 5604.18/5761.07)}\\ &= 5926.99\ \text{Å}, \quad \text{neglecting dispersion.}\end{aligned} \tag{11}$$

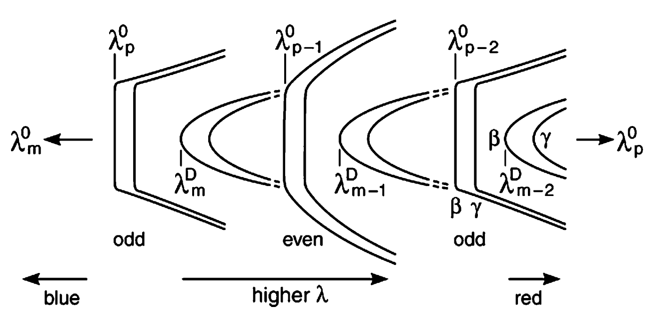

Fig. 1. Schematic representation of the locations and shapes of the FECO in Examples 1 and 2. The flat parts in λ_p^0, λ_{p-1}^0, and λ_{p-2}^0, result from the elastic distortion of the glue which supports the mica [3]. For such a flat contact of mica in air, odd $(\ldots, p, p-2, \ldots)$ fringes always have a similar shape, which is different from that of the even $(\ldots, p-1, p-3, \ldots)$ fringes. Specifically, the place at which the fringes stop being flat and start bending has a bigger discontinuity in its derivative for odd fringes than for even fringes. Also note that since $m > p$, λ_m^D, λ_{m-1}^D, and λ_{m-2}^D are closer together than λ_p^0, λ_{p-1}^0, and λ_{p-2}^0.

If we include dispersion in the refractive index, then we need to know the relation for the refractive index of the fringe (β or γ) that is being measured as a function of λ. For "brownish mica," this is [17]

$$\mu_\beta = 1.5794 + 4.76 \times 10^5/\lambda^2 \quad \text{for } \beta \text{ fringes,} \tag{12}$$

where λ is the wavelength in Å. We first calculate the refractive indices at the three contact wavelengths, and obtain $\mu_\beta(\lambda_p^0) = 1.5946$, $\mu_\beta(\lambda_{p-1}^0) = 1.5937$, $\mu_\beta(\lambda_{p-2}^0) = 1.5929$. Now, using Eq. (5), we obtain

$$\lambda_{p-2}^0 = \frac{1.5929}{1.5946} \frac{5604.18}{\left[1 - 2\left(1 - \frac{1.5937}{1.5946}\frac{5604.18}{5761.07}\right)\right]} = 5927.24\ \text{Å}, \tag{13}$$

which is in excellent agreement with the measured value of 5927.38 ± 0.21 Å, indeed, within the experimental error. Actually, one could use the measured value of λ_{p-2}^0 to determine one of the constants in Eq. (12) or, if we also measured λ_{p+1}^0 or λ_{p-3}^0, to get both constants. Note that it is not necessary to know the value of p, nor does it matter if p is odd or even—the equations are identical. Only for the separation measurements of the three-layer interferometer is it important to know whether p is odd or even (it happened to be odd for this specific example). We also note that this method can be used to calculate the contact wavelength of *any* fringe order, not just λ_{p-2}^0.

2.2. Example 2

Calculating distances D from λ_p^0 and λ_{p-1}^0, and the measured positions λ_m^D and λ_{m-1}^D of any two adjacent fringes of unknown order m and $m-1$.

A measured contact position of a fringe of unknown order p is at $\lambda_p^0 = 5466.969$ Å and that of order $p-1$ is at $\lambda_{p-1}^0 = 5580.093$ Å. The substrate is brownish mica, whose refractive index is given by Eq. (12). The surfaces are well separated and we want to calculate the distance D between the two surfaces. We know that the medium between the two mica substrates has a refractive index of 1.410 [14], and we perform simultaneous measurements of λ_m^D and λ_{m-1}^D and obtain $\lambda_m^D = 5550.223$ Å and $\lambda_{m-1}^D = 5653.46$ Å (see Fig. 1 for the qualitative relative positions of λ_p^0, λ_{p-1}^0, λ_m^D, λ_{m-1}^D in this example). For a quick estimate of D, one can use Eq. (6) to calculate the contact positions of various fringes $p+1, p+2, \ldots$ (see Example 1) and Eq. (1a) to calculate the distances of our three-layer interferometer until $p + (p - m)$ is found. A more accurate approach is to use Eqs. (5) and (6) for the calculation of the positions $p+1, p+2, \ldots,$ as described in Example 1, and Eq. (9) for the distances. In this example we show a case where Eq. (5) is used with Eq. (1a) [14]. Obviously $m \geqslant p$; hence our first guess is $m = p$. Using this guess, we put the values for λ_p^0, λ_{p-1}^0, and λ_m^D (which we guess to be λ_p^D) in the *odd* version of Eq. (1a) and obtain $D = 1363$ Å. We then perform the same calculation using λ_{m-1}^D (which we guess to be λ_{p-1}^D) and the values for λ_{p-1}^0 and λ_{p-2}^0 using the *even* version of Eq. (1a), and obtain $D = 1310$ Å. Since $1363 \neq 1310$, our guess was wrong. Our second guess would be $m = p + 1$ (where m is now even), and we use the even version of Eq. (1a) with the calculated value of λ_{p+1}^0, measured λ_p^0 and λ_m^D (which we guess to be λ_{p+1}^D). We obtain $D = 3288$ Å. The odd version of Eq. (1a) with λ_p^0, λ_{p-1}^0, and λ_{m-1}^D (which we guess to be λ_p^D), now results in $D = 3280$ Å. Since $3288 \cong 3280$, we conclude that $m = p + 1$. The difference of 8 Å is in part because the refractive index of the medium was assumed to be nondispersive, and in part because the mica–silver phase change was also assumed to be nondispersive. We should also note that for $m = p + 2$, we obtain $D = 5217$ Å using the odd version of Eq. (1a) with λ_{p+2}^0, λ_{p+1}^0, and λ_m^D (guessed to be λ_{p+2}^D), and $D = 5248$ Å using the even version of Eq. (1a) with λ_{p+1}^0, λ_p^0, and λ_{m-1}^D (guessed to be λ_{p+1}^D), and we get $D(\lambda_m^D, \lambda_{p+2}^0, \lambda_{p+1}^0) < D(\lambda_{m-1}^D, \lambda_{p+1}^0, \lambda_p^0)$. Indeed, for all guesses above $p+1$ the second estimate is higher than the first, while for all guesses below $p+1$ the reverse is true, and this observation is general. In other words, the separation calculated using a higher order contact fringe couple is bigger than that using a lower order contact fringe couple for $(m-p)$ guesses which are smaller than the real $(m-p)$, whereas the separation calculated using a higher order contact fringe couple is smaller than that using a lower order contact fringe couple for $(m-p)$ guesses which are bigger than the real $(m-p)$. Formulating this would look like this:

$$D\left(\lambda_m^D, \lambda_{p+i}^0, \lambda_{p+i-1}^0\right) > D\left(\lambda_{m-1}^D, \lambda_{p+i-1}^0, \lambda_{p+i-2}^0\right) \quad \text{for } i < (m-p),$$
$$D\left(\lambda_m^D, \lambda_{p+i}^0, \lambda_{p+i-1}^0\right) = D\left(\lambda_{m-1}^D, \lambda_{p+i-1}^0, \lambda_{p+i-2}^0\right) \quad \text{for } i = (m-p),$$
$$D\left(\lambda_m^D, \lambda_{p+i}^0, \lambda_{p+i-1}^0\right) > D\left(\lambda_{m-1}^D, \lambda_{p+i-1}^0, \lambda_{p+i-2}^0\right) \quad \text{for } i > (m-p).$$

The above example uses only $m = p + 1$ ($i = 1$) and would work for most cases only for very small values of $(m-p)$. Thus, in general it is advised to count the number of passing fringes in order to determine $(m-p)$. Theoretically, it should be possible to include the dispersive refractive index of the medium *and* the dispersive phase change at the reflector–substrate interface. However, as we discuss below, incorporating a dispersive phase change, while theoretically desirable, is difficult in practice.

Since the above did not include the dispersive phase change, the similarity obtained for the two calculations of D does not suggest that this is the "error" in the separation, which could be larger. As we shall see, incorporation of the phase change gives a formally accurate solution; however, there are good reasons to use only the substrate dispersive refractive index and neglect the dispersive phase change.

3. Comparing dispersion and phase change

If we consider a normal SFA experiment with mica as a substrate and a silver layer of 550 Å as a reflector, then out of the two corrections to the ideal position of the fringes (substrate refractive index dispersion and substrate–reflector dispersive phase change), the substrate dispersion is usually the bigger contributor, as demonstrated in Fig. 2. In addition to having a smaller effect, the phase change adds complexity and requires the use of phenomenological relations. The above discussion does not neglect phase change completely, but rather uses a constant value for the phase change at $D = 0$ (as explained by Bailey and co-workers [8]), since the wavelength positions measured in contact correspond to the reflector phase changing system that produced them. Thus, for many systems, this constant value correction suffices. However, there are situations in which one would like to incorporate the dispersive phase change.

4. Incorporating the phase change at the substrate–reflector (mica–silver) interface

For better accuracy there is a need to incorporate the phase change at the substrate–reflector interface. We use the idea suggested by Bailey and co-workers [8], and we follow their methodology to incorporate the phase change contribution explicitly. The phase change has the effect of adding (or subtracting) an apparent thickness y to the real thickness, and hence mica of real thickness Y' will, due to the phase change, appear to be of thickness $Y = Y' + y$. Note that y is a function of the wavelength.

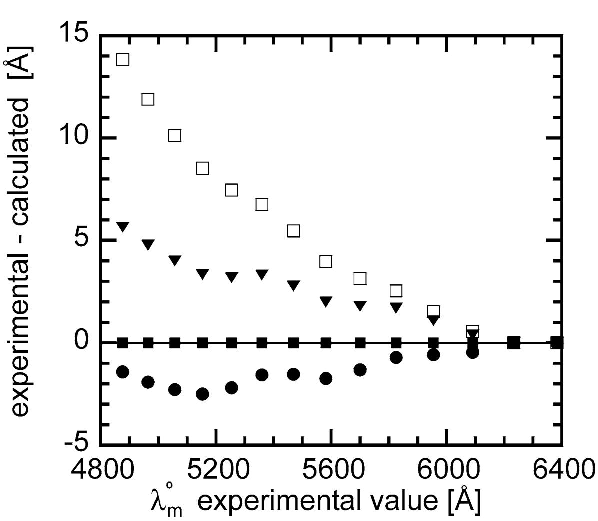

Fig. 2. The difference [Å] between the calculated and measured (experimental) contact wavelengths for three calculations: square—use Eq. (6) (neglecting both dispersion of mica and of mica–silver interface); triangle—use Eq. (5) (neglecting only dispersion of mica–silver interface); circle—use Eq. (20) (considering dispersion both of mica and of mica–silver interface). The filled squares on the vertical line across the 0 of the ordinate correspond to the measured experimental value. All the calculations were done using the two rightmost measured experimental data points.

We start again with the single layer interferometer, and following Bailey and co-workers [8], we can rewrite Eq. (3) as

$$2\mu_p(Y' + y_p) = p\lambda_p^0, \tag{14a}$$

$$2\mu_{p-1}(Y' + y_{p-1}) = (p-1)\lambda_{p-1}^0, \tag{14b}$$

$$2\mu_m(Y' + y_m) = m\lambda_m^0. \tag{14c}$$

Solving Eq. (14) for p, we get

$$\frac{1}{p} = \frac{\lambda_{p-1}^0 - \lambda_p^0 \frac{\mu_{p-1}}{\mu_p}}{\lambda_{p-1}^0 - \mu_{p-1}(y_p - y_{p-1})}, \tag{15}$$

where y_i is the value of y for the wavelength λ_i^0 (that is, $y(\lambda_i^0)$) and it can easily be seen that for a constant (nondispersive) y, Eq. (15) reduces to Eq. (4).

Generally $y(\lambda_i)$ (and particularly $y(\lambda_i^0)$) is a function of the dispersive phase change $\phi(\lambda_i)$,

$$y(\lambda_i) = \phi(\lambda_i)\frac{\lambda_i}{4\pi\mu(\lambda_i)}, \tag{16}$$

where $\phi(\lambda_i)$ was shown by Bailey and co-workers [8] to be given by the expression

$$\phi(\lambda_i) = \arctan\left(\frac{k(\lambda_i)}{n(\lambda_i) - \mu(\lambda_i)}\right) - \arctan\left(\frac{k(\lambda_i)}{n(\lambda_i) + \mu(\lambda_i)}\right), \tag{17}$$

where $k(\lambda_i)$ and $n(\lambda_i)$ are the imaginary and real parts of the complex refractive index of the reflector (usually silver in our case). Tabulated values for $k(\lambda_i)$ and $n(\lambda_i)$ can be found in ordinary handbooks (see, for example, Ref. [18]). In Fig. 2, we used the heuristic relation from Ref. [19], which was obtained from measurements with a silver layer 250–500 Å thick deposited by evaporation:

$$k(\lambda_i) = -2.991902 + 0.001514\lambda_i - 5.770477\lambda_i^2,$$
$$n(\lambda_i) = 0.5. \tag{18}$$

In Ref. [19], silver films of thickness 185–500 Å were measured, and no dependence on film thickness was observed for films thicker than 250 Å. Note that Bailey and co-workers also do not consider the phase change at the mica silver interface to be a strong function of the silver thickness [8]; rather, the silver roughness, which depends on the way of silver deposition, the temperature and pressure, and the duration of the deposition, is the dominant factor in the phase change. Now, the value of $(y_p - y_{p-1})$ can easily be calculated from

$$y_p - y_{p-1} = y(\lambda_p^0) - y(\lambda_{p-1}^0) = \phi(\lambda_p^0)\frac{\lambda_p^0}{4\pi\mu_p^0} - \phi(\lambda_{p-1}^0)\frac{\lambda_{p-1}^0}{4\pi\mu_{p-1}^0}, \tag{19}$$

where $\phi(\lambda_i)$ is given by Eq. (17). This in turn can be substituted into Eq. (15) to calculate $1/p$.

We can now use Eq. (14) to calculate the position of any contact fringe λ_m^0 based on two measured fringes λ_p^0 and λ_{p-1}^0,

$$\lambda_m^0 = \frac{\mu_m}{\mu_p}\frac{\lambda_p^0}{1-(p-m)\left(\frac{\lambda_{p-1}^0-\mu_{p-1}(y_p-y_{p-1})}{\lambda_{p-1}^0-\lambda_p^0\mu_{p-1}/\mu_p}\right)} - \frac{2\mu_m(y_p-y_m)}{\left(\frac{\lambda_{p-1}^0-\mu_{p-1}(y_p-y_{p-1})}{\lambda_{p-1}^0-\lambda_p^0\mu_{p-1}/\mu_p}\right)-(p-m)}, \quad (20)$$

and one can see that for a nondispersive y, Eq. (20) reduces to Eq. 5.

Figure 2 shows the calculated positions of λ_m^0 using Eqs. (20), (5), and (6). Equation (6) is an approximate solution which neglects both mica dispersion and mica–silver phase dispersion; Eq. (5) accounts for the substrate (mica) dispersion but neglects the phase change at the substrate reflector (mica–silver) interface; and Eq. (20) accounts for both substrate and substrate–reflector phase dispersions. We can see that as the equation used is more exact the calculated wavelength position becomes indeed increasingly close to the measured one. Apparently, however, Eq. (20) fails to calculate the exact position of wavelength and does not provide an absolutely accurate solution. This is not surprising for two reasons. One, as noted earlier, is that the relations in Eq. (18) are dependent on the exact experimental conditions that formed the silver layer, which can be different from experiment to experiment. Another reason is that Eq. (20) has two opposite large corrective terms, which eventually result in a small correction. Hence a small inaccuracy in the measurement of the wavelength or in Eq. (18) could result in a rather big error if Eq. (20) were used. (Unlike Eq. (5), where the corrective term merely multiplies an existing term by a small number.)

Thus, relying on literature values for the reflector phase change is problematic since they may use different conditions than the experimental ones, which cannot necessarily be accounted for (e.g., silver roughness at the interface). We therefore suggest measuring the values of $k(\lambda_i)$ and $n(\lambda_i)$ using the known $(m-p)$ values in Eq. (20) for every experiment in which it may be necessary and recalculating the constants using Eq. (18). The only problem with this approach is that unlike the dispersive refractive index, which uses the rather well-established Cauchy relation, the equations for the dispersive phase change are purely heuristic empirical relations, and hence the extrapolation (and perhaps even the interpolation) of the experimental values is not well-founded and could be erroneous.

To complete the picture, we write the equation for the three-layer interferometer that accounts for the phase change. Here again we follow Bailey and co-workers [8], who noted that in Eq. (1) the term $4\pi\mu Y/\lambda_p^D$ can be replaced by

$$4\pi\mu_D Y/\lambda_p^D = \frac{\pi\lambda_p^0}{\lambda_p^D}\left[p\frac{\mu_D}{\mu_0} - \frac{4\mu_D}{\lambda_p^0}\left(y_p - y_p^D\right)\right], \quad (21)$$

where μ_D is $\mu(\lambda_p^D)$, μ_0 is $\mu(\lambda_p^0)$, $y_p^D = y(\lambda_p^D)$, and $y_p = y(\lambda_p^0)$. Substituting p from Eq. (15) into Eq. (21) and substituting that into Eq. (1) would give the correct expression for the three-layer interferometer, which accounts for both the dispersion and the phase change.

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[13] The optical thickness Y which takes into account the phase change of light at the reflector–substrate interface, as opposed to the physical thickness Y', is a function of the wavelength and the thickness of the reflector (silver) layer. Throughout the first part of the paper the approximation of a constant Y is made. Although later in the paper the dispersive Y is accounted for, note that in practice, the use of a constant Y is more useful for most cases.
[14] In principle, one should use the dispersive form of the refractive index for the medium as well; however, from Eq. (10) one sees that the refractive index of the *medium* is only relevant for the measured fringes which, by definition, are all in the visible range. Thus, the error in the dispersion of the medium is limited to the narrow visible range. On the other hand, the refractive index of the *substrate* should be known at the measured (visible) fringe as well as at the contact positions which could be well outside the visible range. Thus, the dispersion of the substrate does need to be known.
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